

NEUTRON STAR KICKS BY THE GRAVITATIONAL TUG-BOAT MECHANISM IN ASYMMETRIC SUPERNOVA EXPLOSIONS: PROGENITOR AND EXPLOSION DEPENDENCE

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ABSTRACT

Asymmetric mass ejection in the early phase of supernova (SN) explosions can impart a kick velocity to the new-born neutron star (NS). For neutrino-driven explosions the NS acceleration was shown to be mainly caused by the gravitational attraction of the anisotropically expelled inner ejecta, while hydrodynamic forces contribute on a subdominant level, and asymmetric neutrino emission plays only a secondary role. Two- and three-dimensional hydrodynamic simulations demonstrated that this gravitational tug-boat mechanism can explain the observed space velocities of young NSs up to more than 1000 km s^{-1} . Here, we discuss how the NS kick depends on the energy, ejecta mass, and asymmetry of the SN explosion, and which role the compactness of the pre-collapse stellar core plays for the momentum transfer to the NS. We also provide simple analytic expressions for the NS velocity in terms of these quantities. Referring to results of hydrodynamic simulations in the literature, we argue why within the discussed scenario of NS acceleration, electron-capture SNe, low-mass Fe-core SNe and ultra-stripped SNe can be expected to have considerably lower intrinsic NS kicks than core-collapse SNe of massive stellar cores. Our basic arguments remain valid also if progenitor stars possess large-scale asymmetries in their convective silicon and oxygen burning layers. Much of our discussion remains on a conceptual and qualitative level and more work is needed on the numerical modeling side to determine the dependences of involved parameters, whose prescriptions will be needed for recipes that can be used to better describe NS kicks in binary evolution and population synthesis studies.

Keywords: supernovae: general — stars: neutron — hydrodynamics — instabilities — neutrinos

1. INTRODUCTION

Neutron stars (NSs) are born with kick velocities of typically $200\text{--}500 \text{ km s}^{-1}$, which is evidenced by the measured proper motions of young radio pulsars (exceeding the break-up velocities of close double-star systems) and by the orbital parameters and spin orientations of NSs in binary systems (e.g., [Harrison et al. 1993](#); [Kaspi et al. 1996](#); [Lyne & Lorimer 1994](#); [Fryer et al. 1998](#); [Lai et al. 2001](#); [Arzoumanian et al. 2002](#); [Chatterjee et al. 2005](#); [Hobbs et al. 2005](#)). Also recently detected hypervelocity stars might originate from disrupted binaries via asymmetric supernovae (SNe) with large NS kicks ([Tauris 2015](#)). These NS kicks could be a consequence of asymmetric explosions (e.g., [Janka & Müller 1994](#); [Burrows & Hayes 1996](#)) or anisotropic emission of the neutrinos that carry away the huge binding energy of the compact star (e.g., [Woosley 1987](#); [Bisnovatyi-Kogan 1993](#); [Fryer & Kusenko 2006](#); [Kusenko et al. 2008](#); [Sagert & Schaffner-Bielich 2008](#)).

The non-radial flow instabilities (convective overturn and the standing accretion shock instability, SASI; [Blondin et al. 2003](#); [Foglizzo 2002](#); [Foglizzo et al. 2006, 2007](#); [Scheck et al. 2008](#)) that develop shortly after core bounce in the postshock accretion layer of collapsing stellar cores, produce mass-ejection asymmetries of the SN

explosion that can be sufficiently large to account for NS kicks of several hundred km s^{-1} with cases reaching up to and even beyond 1000 km s^{-1} . This was demonstrated by two-dimensional (2D) and three-dimensional (3D) hydrodynamic simulations of neutrino-driven SN explosions ([Scheck et al. 2004, 2006](#); [Nordhaus et al. 2010, 2012](#); [Wongwathanarat et al. 2010, 2013](#)), which led to the conclusion, consistent with linear momentum conservation, that the NS must receive a natal kick opposite to the momentum of the SN ejecta.

Seemingly not aware of these theoretical works, [Bray & Eldridge \(2016\)](#) proposed recently, based on their inspiration and interpretation of observational and modeling results of Cassiopeia A, exactly this connection of measured NS kicks and SN explosion asymmetries. Correspondingly, they suggested a direct relationship between the velocity of the compact remnant and the ratio of SN ejecta mass to NS mass.

During the asymmetric ejection of the SN debris, the NS acceleration happens by momentum transfer through hydrodynamic pressure forces and momentum advection in outflows and downflows, by gravitational forces of the anisotropic ejecta on the compact remnant, and by nonspherical neutrino-emission. [Scheck et al. \(2006\)](#) and

Wongwathanarat et al. (2013) found that the anisotropic gravitational interaction as a non-saturating long-range force has significant influence over several seconds and therefore makes the far dominant effect, whereas neutrino emission associated with asymmetric accretion contributes only on a low level. For this reason Wongwathanarat et al. (2013) introduced the term “gravitational tug-boat mechanism” for the physical process that is mainly responsible for the NS recoil associated with the asymmetric ejection of matter in SN explosions: the slowest and usually densest and most massive ejecta “clumps” exert the strongest forces on the NS such that their pull accelerates the compact remnant opposite to the direction of the more powerful explosion. This scenario is consistent with the conservation of the total linear momentum of SN ejecta and NS in the rest-frame of the progenitor star. If high birth kicks of black holes (BHs) are required to explain the spatial distribution of Galactic low-mass X-ray binaries containing BHs (Repetto et al. 2012; Repetto & Nelemans 2015; for counter-arguments, however, see Mandel 2016), the gravitational tug-boat mechanism can offer a scenario in which the high kicks are a consequence of considerable amounts of matter that remain gravitationally bound during the SN explosion and fall back asymmetrically to the compact remnant (Janka 2013). Otherwise, if high natal BH kicks are not required by observations, the operation of the gravitational tug-boat mechanism would mean that BH formation events with significant SN fallback are rare in the Galactic neighborhood, compatible with conclusions drawn from recent theoretical studies of the progenitor-SN connection (Ugliano et al. 2012; Ertl et al. 2016; Sukhbold et al. 2016). Or, alternatively, it could mean that most BHs originate from SNe with very little or rather spherical mass ejection.

The majority of cases simulated by Scheck et al. (2006) and Wongwathanarat et al. (2013) produced NS kicks of several 100 km s^{-1} in good match with the maximum of the observed NS velocity distribution. In the set of 2D models of Scheck et al. (2006), one out of 70 cases developed an estimated NS kick velocity of more than 1000 km s^{-1} . In 3D such extreme velocities have not been obtained yet, but the simulations by Wongwathanarat et al. (2013) were constrained to a small sample of progenitor stars and a set of only 20 models. One of these cases showed a NS kick velocity in excess of 700 km s^{-1} after 3.3 s of post-bounce evolution, with an acceleration of more than 70 km s^{-2} still boosting the NS velocity. Three other cases had NS kick velocities of nearly 600 km s^{-1} at 3.3 s after bounce with ongoing accelerations of up to $>100 \text{ km s}^{-2}$. Velocities above 1000 km s^{-1} , maybe even considerably exceeding this value, seem to be well possible when the incipient explosion develops a large dipolar asymmetry mode, or when a long-lasting phase of asymmetric accretion transfers momentum to the nascent NS, both of which cases did not occur in the limited set of 3D models computed by Wongwathanarat et al. (2013). Although such scenarios are plausible, given the large explosion

asymmetries seen in recent 3D SN models (e.g., Melson et al. 2015a; Lentz et al. 2015; Müller 2016), a direct numerical demonstration of NS kicks beyond 1000 km s^{-1} by the gravitational tug-boat mechanism in 3D SN simulations is desirable.

The papers by Scheck et al. (2006) and Wongwathanarat et al. (2013) investigated different progenitors with explosion energies and asymmetries varying over rather wide ranges, but the small variety of $15 M_{\odot}$ and $20 M_{\odot}$ models did not allow them to illuminate the systematics of NS kicks in dependence on stellar progenitor properties and SN explosion properties. In this work we attempt to take first steps in this direction. Our discussion will remain mostly on a conceptual and qualitative level, focussing on scaling-laws derived here to provide insights into basic factors that determine the NS kicks by the gravitational tug-boat mechanism. We thus intend to prepare the ground for future comparisons of the theoretical predictions with observations and for an improved description of NS kicks in binary evolution and population synthesis studies. For these future goals to become achievable, the parameters appearing in our scaling laws need to be pinned down in their dependences by more elaborate and longer 3D simulations of larger sets of progenitor stars than currently available in the literature. In the present paper we will briefly review the published results and will interpret their meaning and shortcomings.

The paper is structured as follows. In Sect. 2.1 we present simple scaling relations for the NS kick velocity as a function of the mass of neutrino-heated ejecta, explosion energy, and explosion asymmetry, in Sect. 2.2 we will analyse the role of the core structure and compactness of the progenitor stars, and in Sect. 2.3 we will connect the parameters in the scaling laws to simulation results in the literature. Sect. 3 contains a summary and conclusions.

2. ANALYTICAL SCALING RELATIONS

How does the kick velocity of new-born NSs depend on the structure of the progenitor stars and on the characteristic properties of the SN explosions? This question is not only relevant for interpreting observations, e.g. of hypervelocity stars (Tauris 2015), but also, for example, for stellar population studies and for understanding the evolution of binary stars that give birth to double NS systems (e.g. Voss & Tauris 2003; Tauris et al. 2016). In theoretical models for population synthesis a widely used approach to implement the effects of NS natal kicks is based on single-component or multi-component Maxwell-Boltzmann velocity distributions (see, e.g., Voss & Tauris 2003; Dominik et al. 2015), which was questioned recently by Bray & Eldridge (2016). Instead, the latter authors proposed a simple relationship for the NS kick velocity defined as a linear function of the ratio of the SN ejecta mass to the remnant mass.

Though this suggestion seems to work well, it was not sub-

stantiated by any physical explanation in [Bray & Eldridge \(2016\)](#). In the following we will provide corresponding arguments and will, based on our current understanding of neutrino-driven SN explosions and the gravitational tug-boat mechanism for NS acceleration, derive simple scaling relations for the NS kick velocity, v_{NS} , as function of basic parameters that are linked to the ejecta mass of the SN, the mass-ejection asymmetry, and the energy of the explosion. We will also discuss how the efficiency of the NS acceleration mechanism depends on the density profile and compactness of the stellar core above the initial mass cut, from where the SN shock starts its outward expansion. Our discussion will stay mostly on a didactic and qualitative level, because, as we will argue in Sect. 2.3, hydrodynamic SN explosion simulations are needed to determine the values of the parameters that occur in our formulas. The simulation results presently available in the literature, however, only provide crude guidance but do not allow for definitive, finally quantitative conclusions on the general dependences on progenitor and explosion properties. The only exception is a distinct difference between the NS kicks that can be expected for stars near the low-mass end of the SN progenitors and some lighter cases of ultra-stripped SNe on one side, and SNe of progenitors with massive iron cores at collapse on the other side.

2.1. Kick dependence on the explosion properties

Considering only the NS kick associated with anisotropic mass ejection, momentum conservation in the frame of the progenitor star implies that NS and ejecta momenta are equal in value and opposite in direction, i.e.:

$$v_{\text{NS}} = |v_{\text{NS}}| = \alpha_{\text{ej}} P_{\text{ej}} M_{\text{NS}}^{-1}, \quad (1)$$

where M_{NS} is the mass of the NS¹. P_{ej} is defined by the volume integral

$$P_{\text{ej}} = \int_{M_{\text{ej}}} dV \rho |\mathbf{v}|, \quad (2)$$

with M_{ej} being the relevant ejecta mass to be further discussed below. In Eq. (1) we introduce the momentum-asymmetry parameter

$$\alpha_{\text{ej}} = \frac{|\mathbf{P}_{\text{gas}}|}{P_{\text{ej}}}, \quad (3)$$

when the momentum integral of the ejecta gas is calculated as

$$\mathbf{P}_{\text{gas}} = \int_{M_{\text{ej}}} dV \rho \mathbf{v}. \quad (4)$$

¹ While one is tempted to interpret M_{NS} in this equation as the gravitational mass of the NS, the velocity estimate is better in fact if the NS mass is taken to be the baryonic mass, because much of the hydrodynamic recoil momentum is imparted to the NS at a time when the compact remnant has not yet lost a major fraction of its gravitational binding energy by neutrinos. Note, however, that all of our estimates presented in this paper are not on a level of accuracy that requires a very careful distinction between baryonic and gravitational NS mass, which makes a difference of order $\sim 10\%$ only.

By its definition, P_{ej} is related to the kinetic energy of the ejecta: $E_{\text{kin}} = \frac{1}{2} M_{\text{ej}} \bar{v}_{\text{ej}}^2 = \frac{1}{2} P_{\text{ej}}^2 M_{\text{ej}}^{-1}$, where \bar{v}_{ej} is the average ejecta velocity. This yields

$$P_{\text{ej}} = \sqrt{2 E_{\text{kin}} M_{\text{ej}}} = \sqrt{2 f_{\text{kin}} E_{\text{exp}} M_{\text{ej}}}, \quad (5)$$

where in the second expression we introduced the parameter f_{kin} to relate the kinetic energy of the explosion during the relevant time of NS acceleration with the final SN explosion energy E_{exp} .

Both the values of α_{ej} and f_{kin} are time-dependent and need to be determined by multi-dimensional hydrodynamic simulations.

Using Eq. (5) in Eq. (1), we can derive the expression

$$v_{\text{NS}} = \alpha_{\text{ej}} \sqrt{2 f_{\text{kin}} E_{\text{exp}} M_{\text{ej}}} M_{\text{NS}}^{-1} \quad (6)$$

$$= 211 \text{ km s}^{-1} f_{\text{kin}}^{1/2} \frac{\alpha_{\text{ej}}}{0.1} \left(\frac{E_{\text{exp}}}{10^{51} \text{ erg}} \right)^{1/2} \times \left(\frac{M_{\text{ej}}}{0.1 M_{\odot}} \right)^{1/2} \left(\frac{M_{\text{NS}}}{1.5 M_{\odot}} \right)^{-1}. \quad (7)$$

After the NS kick has saturated (typically after a few seconds, see [Scheck et al. 2006](#); [Wongwathanarat et al. 2013](#)), the NS momentum, $P_{\text{NS}} = M_{\text{NS}} v_{\text{NS}}$, and the ejecta momentum, $|\mathbf{P}_{\text{gas}}| = \alpha_{\text{ej}} P_{\text{ej}}$, remain constant, but α_{ej} , E_{kin} , and M_{ej} may still evolve. We therefore consider as the relevant mass of M_{ej} (determining the time when α_{ej} is measured and vice versa) the mass that is accumulated behind the outgoing SN shock until the NS kick asymptotes to its final value. As the shock propagates farther outward and sweeps up more matter from the spherical progenitor star, the ejecta mass increases while the value of the asymmetry parameter decreases, but the product of both quantities remains constant (unless anisotropic fallback modifies the ejecta asymmetry). It is important to note that, if M_{ej} in Eqs. (6) and (7) were to be interpreted as the total ejecta mass of the SN, α_{ej} would have to be measured after the shock-breakout from the surface of the progenitor star, i.e., after the blast wave has accelerated all of the outer stellar layers. This is not very practical in numerical simulations of the explosion mechanism and NS acceleration, which are usually only carried over a few seconds at most.

In the case of neutrino-driven SN explosions, the ejecta mass that is relevant for the NS acceleration is tightly correlated with the amount of matter that is advected through the shock and accreted towards the nascent NS in downflows to be neutrino heated near the gain radius and anisotropically expelled again. Energy absorption from neutrinos lifts this matter to a state of neutral gravitational binding, and the recombination energy released when free nucleons assemble to α -particles and heavy nuclei in the outflow provides a positive contribution of typically $\epsilon \sim (5 \dots 8) \text{ MeV/nucleon} \approx (5 \dots 8) \times 10^{18} \text{ erg g}^{-1}$ to the explosion energy of the SN ([Janka 2001](#); [Scheck et al. 2006](#);

(Marek & Janka 2009; Müller 2015). Ignoring the additional positive energy contributions from explosive nuclear burning in shock-heated ejecta and from the (essentially spherical) neutrino-driven wind that follows the transient post-bounce phase of simultaneous accretion and asymmetric mass (re-)ejection, and also ignoring the negative energy of the gravitational binding of the overlying stellar layers ahead of the SN shock, we can (approximately) write:

$$E_{\text{exp}} \approx \epsilon M_{\text{ej},\nu} = \epsilon \beta_{\nu} M_{\text{ej}}. \quad (8)$$

where $M_{\text{ej},\nu}$ defines the mass of neutrino-heated postshock ejecta, which we relate to the total (expelled) postshock mass by the parameter $\beta_{\nu} \leq 1$: $M_{\text{ej},\nu} = \beta_{\nu} M_{\text{ej}}$. Equation (8) expresses the fact that in the neutrino-driven mechanism the mass of the neutrino-heated ejecta determines the energy of the explosion. Considering ϵ roughly as a constant, Eq. (8) implies a linear relation between E_{exp} and $M_{\text{ej},\nu}$, which is supported by large sets of 2D explosion simulations (Scheck et al. 2006, see figure 9 and Appendix C there; Gessner 2014; Gessner & Janka 2017). Using Eq. (8) with this assumption in Eq. (5), and assuming also $\beta_{\nu} \sim \text{const}$, we obtain the proportionality relations

$$P_{\text{ej}} \propto M_{\text{ej}} \propto E_{\text{exp}}, \quad (9)$$

which are nicely confirmed by results of multi-dimensional SN simulations during the first second(s) of the explosion (Scheck et al. 2006, figure 11; Gessner 2014).

Employing a typical value of $\epsilon \sim 5 \text{ MeV/nucleon}$ in Eq. (8), which implies

$$\frac{\beta_{\nu} M_{\text{ej}}}{0.1 M_{\odot}} \approx \epsilon_5^{-1} \frac{E_{\text{exp}}}{10^{51} \text{ erg}}, \quad (10)$$

where $\epsilon_5 = \epsilon/(5 \text{ MeV/nucleon})$, we can replace M_{ej} in Eq. (7) by E_{exp} to get

$$v_{\text{NS}} = 211 \text{ km s}^{-1} \left(\frac{f_{\text{kin}}}{\epsilon_5 \beta_{\nu}} \right)^{1/2} \left(\frac{\alpha_{\text{ej}}}{0.1} \right) \times \left(\frac{E_{\text{exp}}}{10^{51} \text{ erg}} \right) \left(\frac{M_{\text{NS}}}{1.5 M_{\odot}} \right)^{-1}. \quad (11)$$

Equation (11) is the main result of this paper. It means that the NS kick grows roughly linearly with the explosion energy (or, alternatively, with the relevant ejecta mass, M_{ej} , by means of Eq. 10) and with the explosion asymmetry α_{ej} . Both dependences are easy to understand: a more asymmetric and more powerful explosion is able to impart a larger kick to the NS. The parameters f_{kin} and β_{ν} depend on the SN shock dynamics and therefore on the radial structure of the progenitor star and the evolution stage of the SN explosion. However, the combination of parameters in the factor $f_{\text{kin}}/(\epsilon_5 \beta_{\nu})$ is typically of order unity at the time when the NS kick saturates (since f_{kin} and β_{ν} are of similar magnitude), and its variation is moderated by the square root of this factor in Eq. (11). The momentum asymmetry parameter α_{ej} depends on the stochastic growth of hydrodynamic instabilities

in the postshock layer, which trigger the onset of an asymmetric explosion. Kick velocities in excess of 1000 km s^{-1} require $\alpha_{\text{ej}} \gtrsim 0.5$ for all other factors in Eq. (11) being unity, which is within reach of some published explosion models (e.g., Scheck et al. 2006; Wongwathanarat et al. 2013). Of course, f_{kin} , β_{ν} , ϵ , and α_{ej} could contain hidden dependences on E_{exp} and the progenitor star, which can only be determined by hydrodynamic explosion modeling.

For practical applications of Eqs. (7) or (11), for example in population synthesis studies, it may be preferable to express the NS kick velocity in terms of the total ejecta mass of the SN, $M_{\text{ej,SN}}$ (see Bray & Eldridge 2016), or the NS mass. In this context it is worth to note that on grounds of their semi-analytic model for the SN progenitor-explosion connection, Müller et al. (2016a) suggested loose correlations between the SN explosion energy, the total ejecta mass of the SN, and the (gravitational) NS mass, which they interpreted as compatible with power-law relations deduced from observational analyses (see their figures 9 and 11). The scatter of their model data, however, is large, and the correlations of the mentioned quantities are much less clearly defined than the tight relation between v_{NS} and E_{exp} expressed by Eq. (11).

2.2. Kick dependence on the progenitor compactness

In this section we would like to address the question how the relevant ejecta mass, M_{ej} , is connected to the properties of the core of the progenitor star. M_{ej} as introduced in Sect. 2.1 is the expelled mass that carries the explosion asymmetries, expressed by the asymmetry parameter α_{ej} , during the time when the new-born NS is accelerated. We can therefore write M_{ej} as the difference of the stellar mass enclosed by the SN shock at the time when the NS kick saturates and the initial (baryonic) mass of the compact remnant:

$$M_{\text{ej}} = M_{\text{prog}}(R_0) - M_{\text{NS,i}}, \quad (12)$$

where R_0 is the (average) shock radius at the considered time.

According to our present understanding of the neutrino-heating mechanism, the onset of the explosion is favored when the stalled SN shock enters into the collapsing, oxygen-enriched silicon shell, at which location the entropy per nucleon of the infalling matter exceeds a value of $s = 4 k_{\text{B}}$ per nucleon (Ertl et al. 2016; Sukhbold et al. 2016). For this reason, a reasonable proxy of the initial NS mass is given by

$$M_{\text{NS,i}} \approx M_{\text{prog}}(s = 4), \quad (13)$$

i.e., by the progenitor mass that is enclosed by the radius where the entropy reaches $s = 4 k_{\text{B}}$ per nucleon.

It is more difficult to derive a useful estimate of $M_{\text{prog}}(R_0)$. The NS kick velocity approaches its final value when the acceleration time scale becomes longer than the expansion time scale of the anisotropic ejecta, in which case the gravitational interaction between ejecta and NS become unimportant. This

requirement can be expressed by

$$t_{\text{acc}} = \frac{v_{\text{NS}}}{a_{\text{NS}}} \gg \frac{R_0}{\bar{v}_{\text{ej}}}, \quad (14)$$

where

$$\bar{v}_{\text{ej}} = \sqrt{\frac{2E_{\text{kin}}}{M_{\text{ej}}}} = \sqrt{\frac{2f_{\text{kin}}E_{\text{exp}}}{M_{\text{ej}}}} \quad (15)$$

is the average ejecta velocity and

$$a_{\text{NS}} = \dot{v}_{\text{NS}} \sim \alpha_{\text{ej}} \frac{GM_{\text{ej}}}{R_0^2} \quad (16)$$

is a crude measure of the NS acceleration. Using Eqs. (6), (15), and (16) in Eq. (14) yields a condition for R_0 :

$$R_0 \gg \frac{G}{2f_{\text{kin}}} \frac{M_{\text{ej}}}{E_{\text{exp}}} M_{\text{NS}}. \quad (17)$$

Applying Eq. (8) we thus obtain

$$R_0 \gg \frac{G}{2f_{\text{kin}}\beta_v} \epsilon^{-1} M_{\text{NS}} \quad (18)$$

$$\sim 200 \text{ km } f_{\text{kin}}^{-1} \beta_v^{-1} \epsilon_5^{-1} \left(\frac{M_{\text{NS}}}{1.5 M_{\odot}} \right), \quad (19)$$

where in the second expression we used a representative value of $\epsilon \sim 5 \text{ MeV/nucleon}$ again. Since during the period of NS acceleration the kinetic energy is usually only a minor fraction of the final explosion energy, $f_{\text{kin}} \sim 0.1$, and also β_v drops with increasing shock radius to values around 0.1–0.2, Eq. (19) suggests that typical values of R_0 are beyond 10,000 km. This is compatible with the numerical result that the NS acceleration in neutrino-driven explosions continues on a significant level for several seconds, during which period the average ejecta velocity is a few 1000 km s⁻¹ (Scheck et al. 2006; Wongwathanarat et al. 2010, 2013).

With the estimate of Eq. (19) suggesting a rather generic value of R_0 , Eq. (12) implies that progenitors with more massive cores enclosed by R_0 (i.e., progenitors with bigger values of the core compactness $M_{\text{prog}}(R_0)/R_0$) tend to have larger ejecta masses M_{ej} and therefore higher explosion energies (see Eq. 8). For this reason such progenitors provide more favorable conditions for higher NS kick velocities. We note in passing that progenitors with these core properties also tend to produce more massive NSs. A rough correlation of NS mass and explosion energy for neutrino-driven explosions was indeed reported by Müller et al. (2016a) (see figure 11 there).

The discussion outlined above is rather qualitative and illuminates only basic dependences of the NS kick on the progenitor conditions. We will return to this topic in more detail in Sect. 2.3. In view of the linear relation of Eq. (8), however, the NS kick formula of Eq. (7) translates into Eq. (11) without requirement of any explicit knowledge of M_{ej} .

A similarly qualitative, though interesting, relation can be derived by a different consideration. Nonspherical mass distributions in the postshock medium with anisotropic accre-

tion downflows and buoyant outflows can continue to develop as long as the postshock velocity, v_{pos} , is lower than the local escape velocity (Marek & Janka 2009; Müller 2015; Müller et al. 2016a). When the stellar matter swept up by the outgoing NS shock expands faster than the escape speed, downflows to the NS will be quenched and the spherically symmetric neutrino-driven wind will finally push the asymmetric ejecta away from the NS, heralding the phase when the NS acceleration ceases. A very rough condition when the NS kick is determined can therefore be coined by the relation

$$v_{\text{pos}} \gtrsim v_{\text{esc}}. \quad (20)$$

Here, we can approximately identify v_{pos} with the average expansion velocity of the ejecta behind the shock ($v_{\text{pos}} \sim \bar{v}_{\text{ej}}$ with \bar{v}_{ej} from Eq. 15), and the postshock escape velocity is given by

$$v_{\text{esc}} \sim \sqrt{\frac{2GM_{\text{prog}}(R_0)}{R_0}}. \quad (21)$$

Introducing the dimensionless compactness parameter (O’Connor & Ott 2011)

$$\xi_0 = \frac{M_{\text{prog}}(R_0)/M_{\odot}}{R_0/1000 \text{ km}}, \quad (22)$$

Eq. (20) yields the condition

$$\xi_0 \lesssim \frac{f_{\text{kin}}}{G} \frac{E_{\text{exp}}}{M_{\text{ej}}} \frac{10^8 \text{ cm}}{M_{\odot}} = \frac{f_{\text{kin}}}{G} \beta_v \epsilon \frac{10^8 \text{ cm}}{M_{\odot}}, \quad (23)$$

where we have used Eq. (8) in the second expression on the rhs. Employing again our typical value for ϵ , we obtain

$$\xi_0 \lesssim 3.75 f_{\text{kin}} \beta_v \epsilon_5. \quad (24)$$

Since the compactness is a monotonically falling quantity with increasing radius outside the ONeMg or iron core (because the stellar density decline is steeper than r^{-2} on average), Eq. (24) sets a lower limit to the distance from the center that must be reached by the shock for postshock asymmetries to freeze out and for the NS kick to approach its terminal value.

The values of f_{kin} , β_v , and ϵ vary only moderately between different progenitors during the NS acceleration phase of a few seconds: f_{kin} is around 0.1 (see figure 2 in Ertl et al. 2016 and figure 7 in Sukhbold et al. 2016), and β_v is initially close to unity and later decreases continuously when not all stellar matter swept up by the outgoing shock gets heated by neutrinos. Taking this for granted, Eq. (24) allows, again, to conclude on a clear difference between progenitor stars with a low core compactness and those with high values of the core compactness: large NS kicks are disfavored by low compactness values, because a smaller ejecta mass is neutrino-heated before the shock reaches radii where the inequality condition of Eq. (24) is fulfilled. In particular for low-mass stars, this condition is satisfied already at the onset of the explosion. Such a low compactness implies a fast acceleration of

the explosion and high ejecta velocities (of very little ejecta mass), leaving asymmetries in the postshock layer little time to develop significant dipole amplitudes.

2.3. Discussion of simulation results

Results of 2D and 3D simulations of neutrino-driven explosions including the determination of NS kicks can be found in the literature for constrained sets of progenitor models. [Scheck et al. \(2006\)](#) and [Wongwathanarat et al. \(2013\)](#) investigated stars in the birth-mass range of $15\text{--}20 M_{\odot}$, [Suwa et al. \(2015\)](#) calculated explosions for ultra-stripped SN Ic progenitors (see [Tauris et al. 2015](#)), and [Gessner \(2014\)](#) (also [Gessner & Janka 2017](#)) performed a systematic study for electron-capture SNe at the low-mass end of core-collapse SN progenitors.

Here we will set the results of these papers into the context of our discussion in Sects. 2.1 and 2.2. Although the results are not finally conclusive about possible variations of the relevant parameters (α_{ej} , f_{kin} , β_{ν}) with progenitor and explosion conditions, they can still provide first insights into some systematic trends, which can be understood on grounds of the equations derived above and can help to set constraints on the remaining degrees of freedom.

[Scheck et al. \(2006\)](#) (figures 9, 11, C.1, and C.2 there) as well as [Gessner \(2014\)](#) found very tight linear relations between explosion energy E_{exp} , radial ejecta momentum P_{ej} , and neutrino-heated ejecta mass $M_{\text{ej},\nu}$, confirming the validity of Eq. (9). In fact, the relations for all investigated stellar models with iron cores fall on top of each other, and the data for the ONeMg-core progenitor studied by [Gessner \(2014\)](#) connect continuously to the low-energy end of the Fe-core results. This means that the proportionality factors in the relations of Eq. (9) are indeed independent of the considered progenitor (at least during the computed post-bounce period of one second) as we assumed in some arguments made in Sect. 2.2.

However, in both sets of simulations there is only a mild positive correlation of the NS kick velocity with the explosion energy (see figure 9 in [Scheck et al. 2006](#)), visible by a slight shift of the ensemble distribution toward high-velocity cases for more energetic explosions. This is confirmed by the 3D models of [Wongwathanarat et al. \(2013\)](#), whose results also do not display clear trends with the explosion energy but rather indicate stronger systematic differences between the considered progenitor models (see figure 8 in [Wongwathanarat et al. 2013](#)). One must caution, however, that the absence of an unambiguous scaling with E_{exp} may be a modeling artifact connected to the use of a prescribed neutrino luminosity from the NS core to trigger the onset of the explosion. If this core luminosity is overestimated relative to the accretion luminosity, the detailed dynamics of the explosion and potentially also its asymmetry might be affected in an unrealistic way. This may have been the case for the more energetic explosions in [Scheck et al. \(2006\)](#) and also

for the cases with a less rapidly contacting inner grid boundary to mimic the time-dependence of the shrinking proto-NS radius, as chosen by [Wongwathanarat et al. \(2013\)](#) to minimize hydrodynamic time-step constraints in their 3D simulations. Fully self-consistent calculations (not using a light-bulb contribution to the neutrino emission) will be needed to clarify this aspect.

Moreover, the results of [Scheck et al. \(2006\)](#) and [Wongwathanarat et al. \(2013\)](#) show large case-to-case variations of the NS kick velocity also for models with similar explosion energies. This velocity spread can be understood by a large statistical scatter of the asymmetry parameter α_{ej} as visible in figure 11 of [Scheck et al. \(2006\)](#) and similarly in the results of [Gessner \(2014\)](#). Such variations reflect the stochasticity of the growth of the explosion asymmetries that result from the chaotic interaction of several hydrodynamic instabilities (convective overturn and Rayleigh-Taylor mass motions, SASI activity, Kelvin-Helmholtz vortex motions at shear-flow interfaces) that play a role in the postshock region on the way to the onset of the explosion. The mean value and the width of the α_{ej} distribution exhibit mild trends of decrease with higher explosion energies (see [Scheck et al. 2006](#); [Wongwathanarat et al. 2013](#)), because more powerful explosions tend to develop faster and thus to extenuate the merging of initially higher-mode flow patterns to global asymmetries with dominant low-order spherical harmonics modes. This mild systematic trend as well as the stochastic variations of α_{ej} mask the linear dependence of v_{NS} on E_{exp} expressed by Eq. (11). As mentioned above, the possible influence of the core-neutrino light-bulb luminosity should be kept in mind here. It is not clear how much it might have affected the mean and width of the α_{ej} distributions obtained by [Scheck et al. \(2006\)](#) and [Wongwathanarat et al. \(2013\)](#). It is well conceivable that the statistical distribution of α_{ej} is fairly independent of the explosion energy and the mean value of the NS kick velocity in fully self-consistent simulations should reflect the linear increase of v_{NS} with E_{exp} in Eq. (11) more prominently than visible in the present model sets.

While stochasticity plays an important role in all cases, also the progenitor structure has a clear influence on the NS kick as discussed in Sect. 2.2. Simulating explosions of stellar models in the $15 M_{\odot}$ regime for a wide range of explosion energies between less than 0.2×10^{51} erg and more than 1.7×10^{51} erg, [Scheck et al. \(2006\)](#) found values of α_{ej} from basically zero to about 0.33 with a mean of about 0.10–0.15. The average NS kicks were several 100 km s^{-1} . In contrast, low-mass progenitors with ONeMg cores and iron cores are expected to explode with low energies of at most around 10^{50} erg ([Kitaura et al. 2006](#); [Dessart et al. 2006](#); [Janka et al. 2012](#); [Melson et al. 2015b](#)). Varying the explosion energy for parametric, neutrino-driven explosions of electron-capture SN models in 2D and 3D between $\sim 0.2 \times 10^{50}$ erg and $\sim 1.7 \times 10^{50}$ erg, [Gessner \(2014\)](#) obtained values of α_{ej} ten

times lower than [Scheck et al. \(2006\)](#) (i.e., up to about 0.036) and NS kick velocities of at most a few km s^{-1} (the maximum kick velocities were around 6 km s^{-1}). Both the small explosion energies (or small ejecta masses, see Eq. 9) and the small explosion asymmetries for the low-mass progenitors are responsible for a weak NS acceleration. The small explosion asymmetries are caused by the rapid (quasi-spherical) development of the explosion and the fast expansion of SN shock and ejecta. Both of these effects are favored by the low compactness of the progenitors outside of the ONeMg or Fe-cores, which does not give hydrodynamic instabilities and accretion downdrafts the chance to persist for a long period of time (see the discussion in connection to Eq. 24).

[Suwa et al. \(2015\)](#) simulated explosions of bare CO stars to mimic the progenitors of ultra-stripped SNe in binaries. The compactness values at an enclosed mass of $1.5 M_{\odot}$ for the lower-mass cases of these ultra-stripped models join the very small ones of the low-mass Fe-core and ONeMg-core progenitors, and for all cases they are considerably smaller than the compactness values of pre-collapse stars in the $15\text{--}20 M_{\odot}$ regime. On the basis of our discussion we therefore expect NS kicks for ultra-stripped SNe with iron cores to be intermediate between those of electron-capture SNe and normal Fe-core SNe. Indeed, [Suwa et al. \(2015\)](#) report NS kicks ranging from $\sim 3 \text{ km s}^{-1}$ to about 75 km s^{-1} . This is compatible with the arguments presented here, but can only be taken as suggestive, because [Suwa et al. \(2015\)](#) did not explore large sets of models to account for stochastic variations of the explosion asymmetry.

3. SUMMARY AND CONCLUSIONS

In this paper we discussed how natal NS kicks by the gravitational tug-boat mechanism in asymmetric SN explosions depend on the properties of progenitor stars and explosions. Our approach was mostly on a didactic and conceptual level, referring to published results in the literature, which, however, are not yet conclusive in all aspects.

Our main result is Eq. (11), which coins the kick velocity as a function of the explosion energy, E_{exp} , and of the momentum-asymmetry parameter α_{ej} . By means of Eq. (10) the explosion energy can be replaced by the relevant ejecta mass. The relevant ejecta mass is determined by explosion models as the expelled mass behind the SN shock front at the time when the NS kick asymptotes to its final value and the (time-dependent) value of α_{ej} is measured. It should not be confused with the total ejecta mass of the SN, because the momentum asymmetry for the total SN ejecta is usually not determined by explosion models.

The main parameters, besides the NS mass, that govern the magnitude of the NS kick velocity are therefore the SN explosion energy and the momentum-asymmetry expressed by the parameter α_{ej} . In the neutrino-driven mechanism, according to our present understanding, E_{exp} might loosely correlate with the total SN ejecta mass and the NS mass, but the scatter

of individual cases is considerable ([Müller et al. 2016a](#)).

On grounds of our results we argued that very small NS kick velocities can be expected for stars near the low-mass end of SN progenitors, which possess very dilute envelopes around their degenerate ONeMg or Fe-cores and are expected to explode with very low energies of $\sim 10^{50}$ erg or less. In addition, the rapid expansion of the SN shock prevents the growth of hydrodynamic instabilities that lead to large dipolar asymmetry modes in the ejecta, for which reason α_{ej} remains small, too. The same conclusions can be drawn for ultra-stripped SNe with (nearly) bare metal cores, which should leave behind NSs with low or only moderate kick velocities, provided their core compactness is similarly low as that of the lowest-mass unstripped core-collapse SN progenitors. For low-mass NSs that are born by progenitors with small values of the core compactness near the low-mass end of stars exploding as SNe, we therefore do not only expect lower SN energies (see also figure 11 in [Müller et al. 2016a](#)) but also a tendency to smaller kick velocities.

In contrast, higher average natal NS kicks can be expected for explosions of more massive SN progenitors, whose dense core environments enforce a longer delay of the onset of the explosion, thus permitting the growth of low-mode hydrodynamic instabilities in the neutrino-heated postshock layer. In such a situation, higher explosion asymmetries can be obtained and also much larger amounts of mass are involved in the neutrino-heating process, which favors higher explosion energies. Both effects together lead to much stronger NS kicks.

Our conclusions are supported by larger sets of multi-dimensional hydrodynamic explosion simulations in several works ([Scheck et al. 2006](#); [Wongwathanarat et al. 2013](#); [Suwa et al. 2015](#); [Gessner 2014](#); [Gessner & Janka 2017](#)) and back up hypothetical low-kick scenarios discussed in the literature involving electron-capture and ultra-stripped SNe ([Podsiadlowski et al. 2004](#); [Tauris et al. 2013, 2015, 2016](#)). We emphasize that—in disagreement with arguments in the literature—high NS kick velocities do not require a long shock stagnation phase. The hydrodynamic instabilities in the postshock layer develop within only $\sim 100\text{--}200$ ms after core bounce. A corresponding delay of the shock runaway is therefore sufficient for large-scale explosion asymmetries to become possible. The models of [Scheck et al. \(2006\)](#) and [Wongwathanarat et al. \(2013\)](#) show most efficient NS acceleration for cases where the explosion sets in fairly early after core bounce, but the ejecta expand so slowly that the accretion-downflow and mass-ejection asymmetries still grow afterwards. For such conditions the gravitational pull of these structures on the NS can continue on a high level for a long period of time to efficiently transfer momentum to the nascent NS.

The simulations available in the literature, however, are not finally conclusive with respect to a possible dependence of the momentum-asymmetry parameter α_{ej} on the explosion

energy. The model results exhibit a tendency of showing the highest NS kicks for moderate explosion energies (see figure 8 in [Wongwathanarat et al. 2013](#)), which correlate with rather high neutrino-heated ejecta masses and large values of the ejecta asymmetry α_{ej} , but not very rapid ejecta expansion. The underlying trend of a reduced ejecta asymmetry with higher explosion energies might, however, be a modeling artifact associated with the use of a light-bulb prescription for the neutrino luminosity from the high-density core of the NS, which dominates the accretion luminosity when high-energy explosions are triggered in these parametric simulations. Fully self-consistent SN models are needed to determine the dependence of α_{ej} on E_{exp} . It is well possible that the statistical distribution of α_{ej} is essentially independent of the SN explosion energy for progenitors other than the discussed cases with lowest core compactness. More 3D hydrodynamic explosion models are also needed to determine the exact statistics (means and widths of the distribution functions) of α_{ej} for a wide range of progenitor stars.

Our main result, Eq. (11) with the relation of Eq. (10), has some similarity to a linear ansatz for the functional dependence of the NS kick velocity on the ratio of the SN ejecta mass to the NS mass recently proposed (but not physically explained) by [Bray & Eldridge \(2016\)](#). These authors, however, used the total mass of the SN ejecta, whose exact connection to the NS kick is not well established on the theory side as discussed here. Interestingly, an optimal fit of the two parameters of the linear function to the measured population-integrated NS kick distribution for a subset of pulsar observational data from [Hobbs et al. \(2005\)](#), seems to require a constant floor value of the NS kick velocity of more than 100 km s^{-1} even for vanishing SN mass ejection. [Bray & Eldridge \(2016\)](#) speculate that this effect, if real, might be connected to a small neutrino-emission asymmetry such as it is, for example, associated with the “self-sustained lepton-number emission asymmetry” (LESA) that was re-

cently discovered in 3D SN simulations by [Tamborra et al. \(2014\)](#). Estimates of the NS kick velocity associated with the LEsa asymmetry by [Tamborra et al. \(2014\)](#) are, however, considerably lower than 100 km s^{-1} . But a variety of other neutrino-kick scenarios have been discussed in the literature, mostly assuming non-standard neutrino physics as well as strong magnetic fields in the NS (e.g., [Bisnovatyi-Kogan 1993](#); [Kusenko et al. 2008](#); [Sagert & Schaffner-Bielich 2008](#), and references therein). It has to be seen whether the interesting offset of the NS kick function will survive further investigation of observational data.

We point out that our discussion remains valid also if large-scale perturbations in the convective burning shells of the pre-collapse star play a role for the development and the asymmetry of neutrino-driven explosions (e.g., [Arnett & Meakin 2011](#); [Couch & Ott 2013](#); [Couch et al. 2015](#); [Müller & Janka 2015](#); [Müller et al. 2016b](#); [Müller 2016](#)). In this case the explosion asymmetry, expressed by our parameter α_{ej} , may also depend on the large-scale asphericities in the convective flows of the burning shells (see the results of [Müller 2016](#)), which could thus have an important effect on the NS kick ([Burrows & Hayes 1996](#)). Presently, however, it is unclear for which progenitors and to which extent such pre-collapse perturbations in the convective Si- and/or O-shells have an impact on the SN explosion dynamics and asymmetry, for which reason our discussion can just highlight a potential relevance in principle.

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